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Exponentiated New Modified Weighted Rayleigh Distribution with Application

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ABSTRACT

A three-parameter exponentiated new modified weighted Rayleigh distribution is proposed from an existing work introduced by Elsherpieny et al. (2017) using a modification method. The modification was done by replacing one of the parameters of the existing distribution with two to gain the modified distribution. Several statistical properties of the target distribution are derived and discussed. We apply a real life data set to illustrate the flexibility and potentiality of the distribution and the result reveals that the proposed distribution is better than other known and new distributions considered for the study.

1. INTRODUCTION

In Statistics, Rayleigh distribution is one of the continuous and important distributions which can be applied to many areas like: biology, engineering, health, agriculture and other related sciences. It has two principal parameters namely scale and shape (and in density function of Rayleigh(), where and emanated from Gaussian distribution. While, weighted distributions are useful to adjust the probabilities of the events as observed and recorded Ramadan (2013). Weighted distributions have been in existence for decades for instance, Patil and Rao (1977) developed it and used for survey of their application, Mahfoud and Patil (1982) worked and written essay in honour of C. R. Rao while, Azzalini (1985) expanded his research on a class of distributions which includes the normal ones.

Some years later in literature, it was discovered that many researchers have done tremendous studies on the extension of Azzalini's work to various levels such as weighted Weibull, modified weighted Weibull and beta weighted Weibull includes Shahbaz et al. (2010), Ramadan (2013), Aleem et al. (2013), Badmus and Ikegwu (2013) and Nasiru (2015). Recently, Badmus and Bamiduro (2014) deeply examined on exponentiated weighted Weibull that has better representation than weighted Weibull and Weibull itself. Oguntunde (2015) proposed exponentiated weighted exponential distribution and studied some of its basic statistical properties. Elsherpieny et al. (2017) and Umar et al. (2018) developed exponentiated new weighted Weibull distributions with applications.

There are two motivational factor for the study: the proposed model is better than some of the existing models in literature and can be fitted to various real life data sets since Rayleigh distribution is useful in several areas like: health, biology, Agriculture and other related sciences. By and large, the main aim of the work is to propose a distribution with high flexibility and wide range applicability from

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existing distribution in literature. Finally, the rest of the article is organized as follows: In section 2, modified new weighted Rayleigh is defined. Exponentiated new modified weighted Rayleigh (ENMWR) distribution is defined and discussed in section 3. In section 4, various properties are derive. The proposed model is illustrated using a real data set to show its flexibility in section 5 and we conclude in section 6.

MATERIAL AND METHODS

The New Modified weighted Rayleigh (NMWR) Distribution

This section presents the density, distribution, reliability and hazard function of the NMWR distribution generated from a new weighted Weibull (NWW) distribution investigated by Nasiru (2015). The modification is based on the definition given in weighted Weibull and modified weighted Weibull distribution (i.e for any and, this implies that both the variable and the parameters can take any integer value greater than zero) Mahdy and Aleem et al. (2013). Badmus et al. (2016) introduced generalized modified weighted Weibull (GMWW) distribution and Badmus et al. (2017) upgraded modified weighted Weibull (MWW) distribution to a beta modified weighted Rayleigh (BMWR) distribution where two more additional shape parameters were introduced for flexibility of the model.

Suppose $F(k)$ is the distribution function (cdf), $f(k)$ is the density function (pdf), $Rel(k)$ is the reliability function and $haz(k)$ is the hazard function exist as the MNWR distribution is defined as

$F(k) = (k; \beta, \theta, \alpha) = 1 - e^{-\beta(1+\alpha^2)k^\theta}$,
for any $k, \beta, \alpha > 0$ and $\theta = 2$ Mahdy (2013),
Aleem et al. (2013) and Elsherpieny et al. (2017)

$$F(k) = (k; \beta, 2, \alpha) = 1 - e^{-\beta(1+\alpha^2)k^2} \quad (1)$$

Then, the associating pdf is given by

$$f(k) = (k; \beta, 2, \alpha) = \frac{dF(k)(k; \beta, 2, \alpha)}{dk},$$

$$f(k) = (k; \beta, 2, \alpha) = (1 + \alpha^2)2\beta k e^{-\beta(1+\alpha^2)k^2} \quad (2)$$

$k, \beta, \alpha > 0$ and $\theta = 2$. Where β and α are scale and shape parameters. Hence, the

reliability function is given by

$$Rel(k) = 1 - F(k) = 1 - [1 - e^{-\beta(1+\alpha^2)k^2}] = e^{-\beta(1+\alpha^2)k^2} \quad (3)$$

and the hazard function of the NMWR is given by

$$haz(k) = \frac{f(k; \beta, 2, \alpha)}{Rel(k; \beta, 2, \alpha)} = \frac{(1+\alpha^2)2\beta k e^{-\beta(1+\alpha^2)k^2}}{e^{-\beta(1+\alpha^2)k^2}} \quad (4)$$

$k, \beta, \alpha > 0$ and $\theta = 2$.

The exponentiated new modified weighted rayleigh (enmwr) distribution

The pdf of ENMWR distribution is derived using (1) and (2) to obtain (5) as follows:

$$g_{ENMWR}(k) = (k; \beta, 2, \alpha, c)$$

$$= c(k; \beta, 2, \alpha, c) \cdot [(k; \beta, 2, \alpha, c)]^{c-1}$$

$$g_{ENMWR}(k) = (k; \beta, 2, \alpha, c)$$

$$= c(1 + \alpha^2)2\beta k e^{-\beta(1+\alpha^2)k^2} [1 - e^{-\beta(1+\alpha^2)k^2}]^{c-1}$$

$k, \beta, \alpha, c > 0$ and $\theta = 2$ (5)

where, c is an additional shape parameter. Meanwhile, the associating cdf of the ENMWR is given by

$$G_{ENMWR}(k) = [k; \beta, 2, \alpha, c]^c$$

$$= [1 - e^{-\beta(1+\alpha^2)k^2}]^c$$

$k, \beta, \alpha, c > 0$ and $\theta = 2$ (6)

Therefore, it can be noticed that taking the limit in (6) when $k \rightarrow \infty$ and $k \rightarrow -\infty$ as

$$\lim_{k \rightarrow \infty} G_{ENMWR}[k; \beta, 2, \alpha, c]^c$$

$$= \lim_{k \rightarrow \infty} G_{ENMWR}[1 - e^{-\beta(1+\alpha^2)k^2}]^c$$

$$\lim_{k \rightarrow \infty} G_{ENMWR}(k) = 1$$

and

$$\lim_{k \rightarrow -\infty} G_{ENMWR}[k; \beta, 2, \alpha, c]^c$$

$$= \lim_{k \rightarrow -\infty} G_{ENMWR}[1 - e^{-\beta(1+\alpha^2)k^2}]^c$$

$$\lim_{k \rightarrow -\infty} G_{ENMWR}(k) = 0$$

Then,

$$\begin{aligned}\lim_{k \rightarrow \infty} G_{ENMWR}(k) &= 1 \\ \lim_{k \rightarrow \infty} G_{ENMWR}(k) &= 1, \lim_{k \rightarrow -\infty} G_{ENMWR}(k) = 0; \\ 0 &\leq G_{ENMWR}(k) \leq 1 \\ Rel_{ENMWR}(k) &= 1 - [k; \beta, 2, \alpha, c]^c \\ &= 1 - [1 - e^{-\beta(1+\alpha^2)k^2}]^c \\ &= [e^{-\beta(1+\alpha^2)k^2}]^c \\ k, \beta, \alpha, c &> 0 \text{ and } \theta = 2 \\ (7)\end{aligned}$$

Some Known and New Reduced Distributions Emanate from the Proposed Distribution

There are some known distributions in literature and new distributions emanate as special cases from ENMWR distribution such as: Modified weighted Rayleigh (MWR) distribution when $\beta=c=1$; ((Aleem et al. (2013) and Badmus et al. (2017)), Modified weighted Exponential (MWE) distribution when $\alpha=c=1$; ((Aleem et al. (2013) and Badmus et al. (2016)), Modified Exponentiated Weibull Rayleigh (MEWR) distribution when $\alpha=1$; New and Modified Exponentiated weighted Rayleigh (MEWR) distribution when $\beta=1$; New

$$\begin{aligned}Haz_{ENMWR}(k) &= \frac{g_{ENMWR}(k)(k; \beta, 2, \alpha, c)}{Rel_{ENMWR}(k)(k; \beta, 2, \alpha, c)} \\ &= \frac{c(1 + \alpha^2)2\beta k e^{-\beta(1+\alpha^2)k^2} [1 - e^{-\beta(1+\alpha^2)k^2}]^{c-1}}{[e^{-\beta(1+\alpha^2)k^2}]^c} \\ k, \beta, \alpha, c &> 0 \text{ and } \theta = 2 \\ (8)\end{aligned}$$

Quantile Function and Second Quantile (2nd Qut)

The quantile function of the ENMWR distribution is denoted by $Q_{-MENWR}^{(u)}$ and is given by

$$\begin{aligned}Q_{ENMWR}(u) &= G_{ENMWR}^{-1}(u) \\ G_{MENWR}(k) &= u \Rightarrow k = G_{ENMWR}^{-1}(u).\end{aligned}$$

Therefore, using the expression (6) above, we get

$$\begin{aligned}G_{ENMWR}(k) &= [1 - e^{-\beta(1+\alpha^2)k^2}]^c \\ u &= [1 - e^{-\beta(1+\alpha^2)k^2}]^c \\ u^{\frac{1}{c}} &= [1 - e^{-\beta(1+\alpha^2)k^2}] \\ 1 - u^{\frac{1}{c}} &= \end{aligned}$$

$$\begin{aligned}1 - e^{-\beta(1+\alpha^2)k^2} \\ \ln\left(1 - u^{\frac{1}{c}}\right) &= \ln(1 - e^{-\beta(1+\alpha^2)k^2}) \\ &\Rightarrow \ln\left(\frac{1}{1 - u^{\frac{1}{c}}}\right) \\ &= \beta(1 + \alpha^2)k^2 \\ Q_{ENMWR}(u) &= \left[\frac{\ln\left(\frac{1}{1 - u^{\frac{1}{c}}}\right)}{\beta(1 + \alpha^2)}\right]^{0.5} \\ (9)\end{aligned}$$

Then, second quantile can be obtained by setting $u = \frac{1}{2}$ in (9) and is given (10) below

$$2nd Q_{ENMWR}(u) = \left[\frac{\ln\left(\frac{1}{1 - (\frac{1}{2})^{\frac{1}{c}}}\right)}{\beta(1 + \alpha^2)}\right]^{0.5} \quad (10)$$

Moment and Generating Function

If a random variable (r.v) K has the ENMWR distribution and the r-th non moment is given by

$$\begin{aligned}\mu'_r &= \left[\frac{1}{\beta(1 + \alpha^2)}\right]^{\frac{r}{2}} c \sum_{v=0}^{\infty} (-1)^v \binom{c-1}{v} \frac{1}{(v+1)^{\frac{r}{2}+1}} \Gamma\left(\frac{r}{2} + 1\right) \\ (11)\end{aligned}$$

Elsherpiency et al. (2017). Also, by letting $r = 1, \dots, 4$, we have the following:

$$\begin{aligned}\mu'_1 &= E(k) \\ &= \left[\frac{1}{\beta(1 + \alpha^2)}\right]^{\frac{1}{2}} c \sum_{v=0}^{\infty} (-1)^v \binom{c-1}{v} \frac{1}{(v+1)^{\frac{3}{2}}} \cdot \frac{\sqrt{\pi}}{2} \\ \mu'_2 &= E(k^2) \\ &= \left[\frac{1}{\beta(1 + \alpha^2)}\right] c \sum_{v=0}^{\infty} (-1)^v \binom{c-1}{v} \frac{1}{(v+1)^2} \cdot (1) \\ \mu'_3 &= E(k^3) \\ &= \left[\frac{1}{\beta(1 + \alpha^2)}\right]^{\frac{3}{2}} c \sum_{v=0}^{\infty} (-1)^v \binom{c-1}{v} \frac{1}{(v+1)^{\frac{5}{2}}} \cdot \frac{3\sqrt{\pi}}{4} \\ \mu'_4 &= E(k^4) \\ &= \left[\frac{1}{\beta(1 + \alpha^2)}\right]^2 c \sum_{v=0}^{\infty} (-1)^v \binom{c-1}{v} \frac{1}{(v+1)^3} \cdot (2)\end{aligned}$$

Similarly, the $var(k) = E(k^2) - [E(k)]^2 = \mu'_2 - (\mu'_1)^2$

Skewness (SC) and Kurtosis Coefficient (KC)

The measures of skewness and kurtosis coefficient are given by

$$SC_{ENMWR}(k) = \frac{[E(k^3)]^2}{[E(k^2)]^3} = \frac{(\mu'_3)^2}{(\mu'_2)^3}$$

and the kurtosis

$$KC_{ENMWR}(k) = \frac{E(k^4)}{[E(k^2)]^2} = \frac{\mu'_4}{(\mu'_2)^2}$$

Order Statistics (OS)

Suppose the R.Vs $K_1, K_2, \dots, K_n$ are called the OS of the sample $K_1, K_2, \dots, K_n$ with pdf of the p th order statistics, $K_{(p)}$ is given as:

$$f_{p:n}(k) = \frac{n!}{(p-1)!(n-p)!} g_{ENMWR}(k) [G_{ENMWR}(k)]^{p-1} [1 - G_{ENMWR}(k)]^{n-p} \text{ for } p = 1, \dots, n \quad (12)$$

Furthermore, the density function of the p th OS is written as:

$$f_{p:n}(k) = \frac{n!}{(p-1)!(n-p)!} \cdot c(1 + \alpha^2) 2\beta k e^{-\beta(1+\alpha^2)k^2} [1 - e^{-\beta(1+\alpha^2)k^2}]^{c-1} \cdot [1 - e^{-\beta(1+\alpha^2)k^2}]^c]^{p-1} \cdot [1 - e^{-\beta(1+\alpha^2)k^2}]^{n-p} \quad (13)$$

Again, the pdf for both smallest and largest OS $K_{(1)}$ and $K_{(n)}$ are given by

$$f_{1:n}(k) = n \cdot c(1 + \alpha^2) 2\beta k e^{-\beta(1+\alpha^2)k^2} [1 - e^{-\beta(1+\alpha^2)k^2}]^{c-1} [1 - [1 - e^{-\beta(1+\alpha^2)k^2}]^c]^{n-1} \quad (14)$$

and

$$f_{n:n}(k) = n \cdot c(1 + \alpha^2) 2\beta k e^{-\beta(1+\alpha^2)k^2} [1 - e^{-\beta(1+\alpha^2)k^2}]^{c-1} [1 - e^{-\beta(1+\alpha^2)k^2}]^c]^{n-1} \quad (15)$$

Maximum Likelihood Estimation (MLE)

Here, we are able to established the estimation of the parameters of the proposed distribution with the help of the method of MLEs. Moreover, suppose a random sample of size n consists $k_1, k_2, \dots, k_n$ from the ENMWR distribution, then

$$g_{ENMWR}(k; \beta, \alpha, c) = c(1 + \alpha^2) 2\beta k e^{-\beta(1+\alpha^2)k^2} [1 - e^{-\beta(1+\alpha^2)k^2}]^{c-1}$$

Hence, the likelihood function of the density function above is given by

$$LL(k; \beta, 2, \alpha, c) = \prod_{i=1}^n g_{ENMWR}(k; \beta, 2, \alpha, c)$$

$$= \prod_{i=1}^n c(1 + \alpha^2) 2\beta k e^{-\beta(1+\alpha^2)k^2} [1 - e^{-\beta(1+\alpha^2)k^2}]^{c-1}$$

$$= c^n (1 + \alpha^2)^n 2^n \beta^n (\prod_{i=1}^n k) (\prod_{i=1}^n e^{-\beta(1+\alpha^2)k^2}) (\prod_{i=1}^n [1 - e^{-\beta(1+\alpha^2)k^2}]^{c-1}) \quad (16)$$

By taking the logarithm in (16), we obtain

$$\begin{aligned} \ln LL(k; \beta, 2, \alpha, c) &= n \ln c + n \ln(1 + \alpha^2) + n \ln \beta \\ &+ n \ln 2 \\ &+ \sum_{i=1}^n \ln k_i - \beta(1 + \alpha^2) \sum_{i=1}^n k_i^2 + \\ &+ (c-1) \sum_{i=1}^n \ln(1 - e^{-\beta(1+\alpha^2)k_i^2}) \end{aligned} \quad (17)$$

Also, taking first and second order partial derivatives with respect to the parameters (β, c, α) and equating to 0 as follows:

$$\begin{aligned} \frac{\partial \ln LL}{\partial \hat{\beta}} &= \frac{n}{\beta} - (1 + \alpha^2) \sum_{i=1}^n k_i^2 \\ &+ (c-1) (1 + \alpha^2) \sum_{i=1}^n k_i^2 \left[\frac{e^{-\beta(1+\alpha^2)k_i^2}}{1 - e^{-\beta(1+\alpha^2)k_i^2}} \right] \\ &= 0 \\ \frac{\partial \ln LL}{\partial \hat{c}} &= \frac{n}{c} + \sum_{i=1}^n \ln(1 - e^{-\beta(1+\alpha^2)k_i^2}) = 0 \end{aligned}$$

$$\begin{aligned}\frac{\partial \ln LL}{\partial \hat{\alpha}} &= \frac{2\alpha n}{1 + \alpha^2} \\ &\quad - 2\alpha\beta \sum_{i=1}^n k_i^2 \\ &\quad + (c \\ &\quad - 1)2\alpha\beta \sum_{i=1}^n k_i^2 \frac{e^{-(1+\alpha^2)\beta k_i^2}}{(1 - e^{-(1+\alpha^2)\beta k_i^2})^2} \\ &= 0 \\ \frac{\partial^2 \ln LL}{\partial \hat{\beta}^2} &= \frac{n}{\beta^2} \\ &\quad - (c \\ &\quad - 1) \sum_{i=1}^n \frac{[(1 + \alpha^2)k_i^2]}{(1 - e^{-(1+\alpha^2)\beta k_i^2})^2} \cdot (1 \\ &\quad - e^{-(1+\alpha^2)\beta k_i^2}) \cdot e^{-(1+\alpha^2)\beta k_i^2} \\ &\quad (- (1 + \alpha^2)k_i^2) \\ &\quad - \left\{ e^{-(1+\alpha^2)\beta k_i^2} \cdot e^{-(1+\alpha^2)\beta k_i^2} \cdot ((1 \right. \\ &\quad \left. + \alpha^2)k_i^2) \right\} = 0 \\ \frac{\partial^2 \ln LL}{\partial \hat{\alpha}^2} &= - \frac{\frac{\partial^2 \ln LL}{\partial \hat{c}^2} = \frac{n}{c^2} = 0}{\{(1 + \alpha^2)2n\} + \{4n\alpha^2\}} \\ &\quad + 2\beta \sum_{i=1}^n k_i^2 - \{(c - 1)2\beta\alpha\}.\end{aligned}$$

$$\begin{aligned}&\left\{ \sum_{i=1}^n k_i^2 \frac{(1 - e^{-(1+\alpha^2)\beta k_i^2}) \cdot (e^{-(1+\alpha^2)\beta k_i^2}) \cdot (-2\alpha\beta k_i^2)}{(1 - e^{-(1+\alpha^2)\beta k_i^2})^2} \right. \\ &\quad \left. - \frac{(e^{-(1+\alpha^2)\beta k_i^2}) \cdot (e^{-(1+\alpha^2)\beta k_i^2}) \cdot (2\alpha\beta k_i^2)}{(1 - e^{-(1+\alpha^2)\beta k_i^2})^2} \right\} \\ &\quad - \left\{ \sum_{i=1}^n k_i^2 \left(\frac{e^{-(1+\alpha^2)\beta k_i^2}}{(1 - e^{-(1+\alpha^2)\beta k_i^2})} \right) \cdot (c - 1)2\beta \right\} \\ &= 0 \\ \frac{\partial \ln LL}{\partial \hat{c} \partial \hat{\alpha}} &= \sum_{i=1}^n \frac{e^{-(1+\alpha^2)\beta k_i^2} \cdot 2\alpha\beta k_i^2}{(1 - e^{-(1+\alpha^2)\beta k_i^2})} = 0 \\ \frac{\partial \ln LL}{\partial \hat{c} \partial \hat{\beta}} &= \sum_{i=1}^n \frac{e^{-(1+\alpha^2)\beta k_i^2} \cdot (1 + \alpha^2)k_i^2}{(1 - e^{-(1+\alpha^2)\beta k_i^2})} = 0\end{aligned}$$

and

$$\begin{aligned}\frac{\partial \ln LL}{\partial \hat{\beta} \partial \hat{\alpha}} &= -2\alpha \sum_{i=1}^n k_i^2 \\ &\quad + (c \\ &\quad - 1) \sum_{i=1}^n \frac{1}{(1 - e^{-(1+\alpha^2)\beta k_i^2})^2} \cdot \left\{ (1 \right. \\ &\quad \left. - e^{-(1+\alpha^2)\beta k_i^2}) \cdot (e^{-(1+\alpha^2)\beta k_i^2}) \right\}.\end{aligned}$$

$$\begin{aligned}2\alpha k_i^2 + (1 + \alpha^2)k_i^2 \cdot (e^{-(1+\alpha^2)\beta k_i^2}) \cdot (2\alpha\beta k_i^2) \\ - \left\{ (e^{-(1+\alpha^2)\beta k_i^2}) \cdot (1 + \alpha^2)k_i^2 \right. \\ \left. \left\{ (e^{-(1+\alpha^2)\beta k_i^2}) \cdot (2\alpha\beta k_i^2) \right\} \right\} = 0\end{aligned}$$

Fisher Information (FI) Matrix Derive on log and log_q

FI matrix is an instrument/tool to prepare for variances. In this work, the FI matrix used is given by

$$E \left[\left(\frac{\partial u}{\partial \vartheta} \right) \left(\frac{\partial u}{\partial \vartheta} \right)^T \right] = \left[\frac{\partial^2 u}{\partial \vartheta \partial \vartheta^T} \right]$$

The variances of $\mathfrak{S}$, i.e var ($\mathfrak{S}$) provides from FI matrix based on is

$$G = n \begin{bmatrix} E_{\beta\beta}^2 & E_{\beta c} & E_{\beta\alpha} \\ E_{c\beta} & E_{cc}^2 & E_{c\alpha} \\ E_{\alpha\beta} & E_{\alpha c} & E_{\alpha\alpha}^2 \end{bmatrix}$$

where, n stands for sample size, E : the integral for partial derivatives $\log(ll)$ of taken over pdf $g(k, \vartheta)$, parameters β, c and α in the subscript of E are the second partial derivatives. Villa and Cankaya (2020), pp 24.

RESULTS AND DISCUSSION

Application

Here, we validate the application of the exponentiated new modified weighted Rayleigh distribution with lifetime data of 20 electronic components studied which used by Elsherpieny et al. (2017), page 50 (four-parameter exponentiated new weighted Weibull distribution). Furthermore, several known distributions are fitted to this data as earlier mentioned above and the results compared in Table 1 below:

DISCUSSION AND CONCLUSION

In this study, we were able to modified the existing four parameters (the exponentiated new weighted Weibull distribution) by equating one of the parameters i.e ($\theta=2$) (to became exponentiated new modified weighted Rayleigh distribution. In addition, we also examined on the statistical properties of the MENWR distribution for instance, quantile, moments and generating

function, order statistics, estimation of parameters and model selection criteria such as AIC, BIC, CAIC, HQIC and their p-values using R-programming for data analysis presented in this paper.

CONCLUSION

However, results from the estimated parameters, goodness of fit and model performance in Tables

Table 1: Estimates, Standard errors (.), and Statistics

Models	Estimates	W	A	KS
ENMWR (Proposed)	0.45390 (0.11769) 0.96411 (12.14493) 2 0.05232 (0.63478)	0.04163	0.25540	0.11743 0.91580 ^p
EMWR (New)	1.06811 (0.23884) 1.52934e-05 (0.07317) 2 1	0.02244	0.17203	0.50670 2.779e-05 ^p
EME (New)	0.45394 (0.11770) 0 2 0.10095 (0.03760)	0.04162	0.25540	0.11745 0.91590 ^p
MWR (Developed by Aleem <i>et al.</i> 2013, Mahdy 2013 and Badmus <i>et al.</i> 2017)	1 1.00893 (13.76033) 2 0.08686 (1.19508)	0.04065	0.25103	0.19648 0.37420 ^p
ENWW (Developed by Elsherpieny <i>et al.</i> (2017))	0.23624 (0.14121) 0.00229 (55.16131) 3.41971 (1.59281) 0.00932 (0.02500)	0.03223	0.21390	0.10319 0.96860 ^p

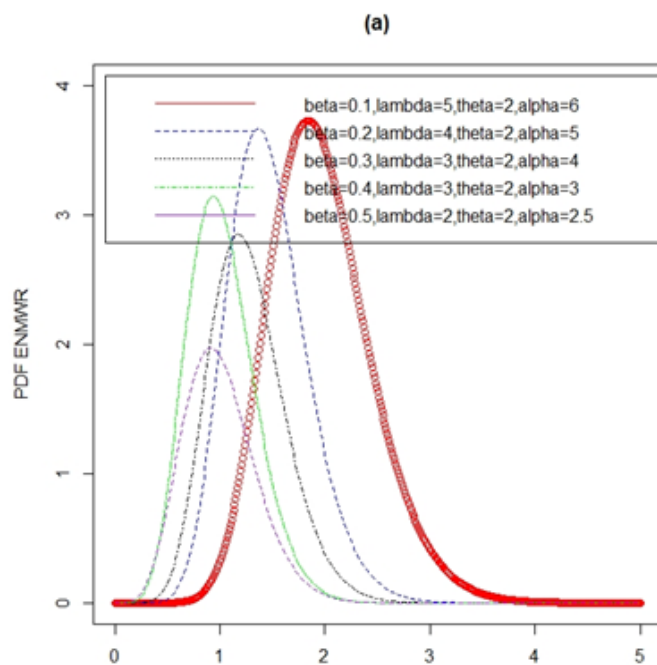


Figure 1: The PDF Plot of ENMWR Distribution for various values

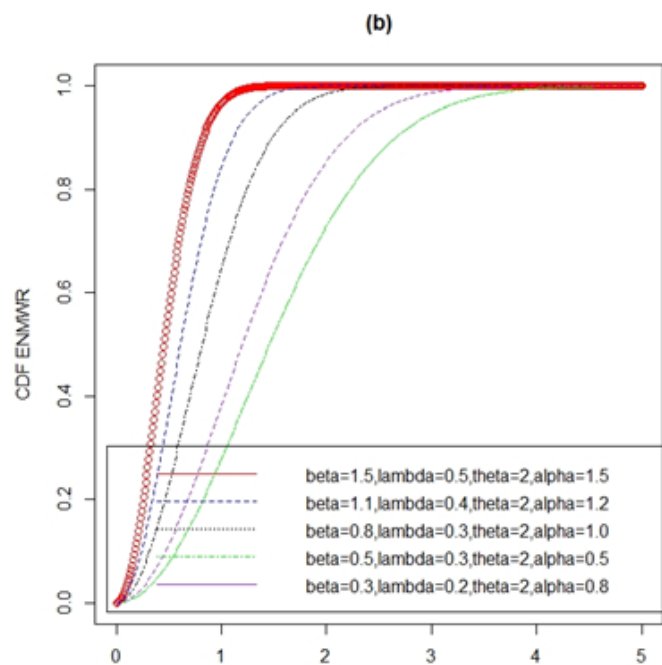


Figure 2: The CDF Plot of ENMWR Distribution for different values

1 and 2 above show that the exponentiated new modified Weighted Rayleigh distribution has appeared to be superior than other distributions considered here.

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Table 2: Model Performance

Model	Model Selection Criteria					Ranks
	-2LogL	AIC	CAIC	BIC	HQIC	
ENMWR	31.80722	67.61443	68.32031	69.60589	68.00318	1 st
EMWR	96.75673	197.5135	198.2193	199.5049	197.9022	5 th
EME	31.80722	69.61443	71.11443	72.60163	70.19756	2 nd
MWR	37.51925	79.03849	79.74437	81.02996	79.42725	4 th
ENWW	31.47063	70.94125	73.60792	74.92418	71.71876	3 rd

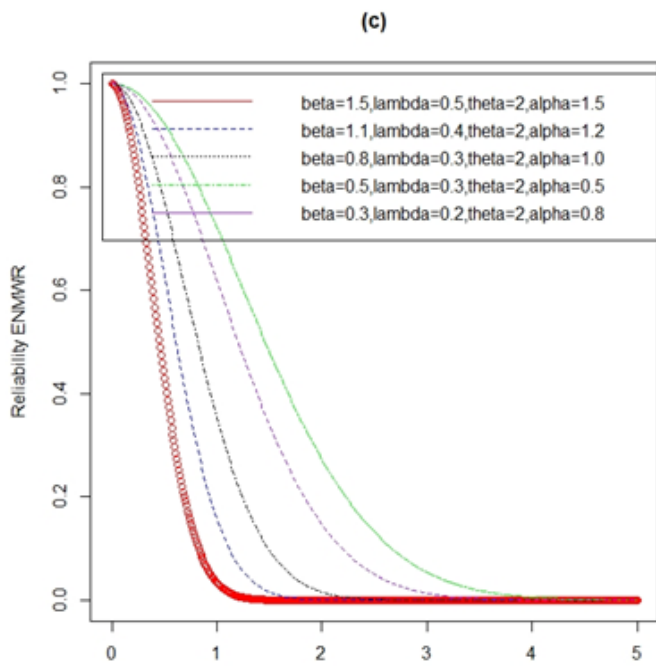


Figure 3: The Reliability Function of ENMWR Distribution for several values

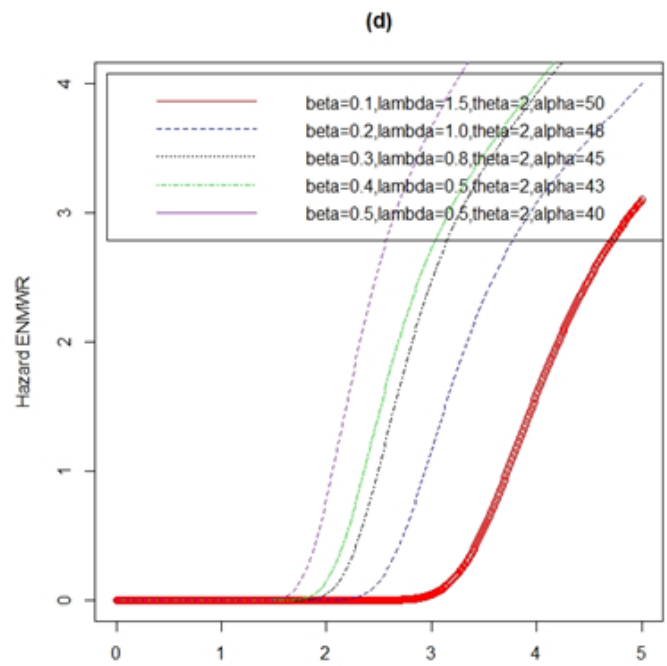


Figure 4: The Reliability Function of ENMWR Distribution for various values

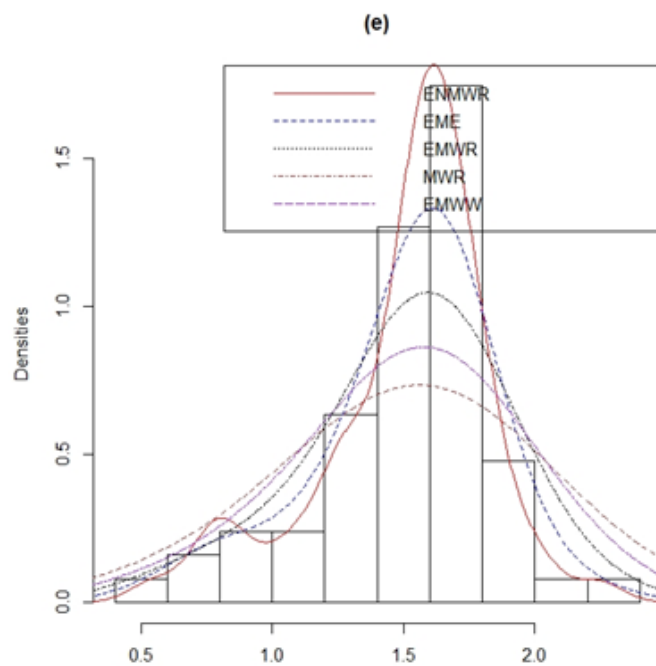


Figure 5: PDF plots for the fitted Distributions for the Data set

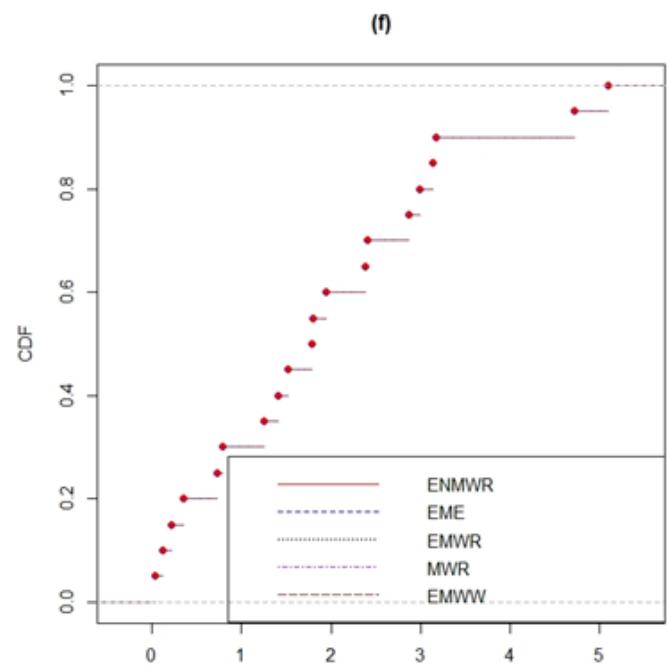


Figure 6: CDF plots for the fitted Distributions for the Data set

Author's Contribution

This work was carried out in collaboration between both authors. Badmus initiated the study, performed the statistical analysis, wrote the protocol and first draft of the manuscript. Authors Badmus and Faweya searched for the literatures used and did the analyses of the study. Then, both authors read, revised and approved the final manuscript submitted for publication.

Conflict Of Interest Statement

The authors declare no conflict of interest regarding the publication of this manuscript.

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